Decision theoretic approach to evaluating and selecting pre-season management plans

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1 Elements of decision theory and relevance to pre-season planning

The three basic components of the decision theory perspective are:

1. ($\mathcal{A}$) Action Space: the set of choices.

   e.g., different pre-season management plans

2. (Θ) States of Nature: the set of possible 'environmental conditions' representing the randomness or uncertainty in the system in which a decision must be made

   e.g., survival rates, harvest rates, migration pattern

3. ($\mathcal{L}$) Loss Function: a function of the action taken and the particular state of nature

   e.g., for a particular management plan (a) and particular set of environmental conditions and model parameters (θ), L(θ,a) could be the sum of squared differences between actual escapements and target escapements.

In the simplest case of a discrete finite set of actions, say q, and states of nature, say p, the loss table (or its negative, the payoff or utility table) is a convenient summary. Suppose the probability of each state of nature is known, too.

		Actions			
States	Probab.	a_1	a_2	$\ldots$	a_q
θ_1	$\Pr(\theta_1)$	$\mathrm{L}(\theta_1, a_1)$	$\mathrm{L}(\theta_1, a_2)$	$\ldots$	$\mathrm{L}(\theta_1, a_q)$
θ_2	$\Pr(\theta_2)$	$\mathrm{L}(\theta_2, a_1)$	$\mathrm{L}(\theta_2, a_2)$	$\ldots$	$\mathrm{L}(\theta_2, a_q)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ldots$
θ_p	$\Pr(\theta_p)$	$\mathrm{L}(\theta_p, a_1)$	$\mathrm{L}(\theta_p, a_2)$	$\ldots$	$\mathrm{L}(\theta_p, a_q)$

Two well-known *decision rules* are minimax and Bayes rule. Minimax says choose the action which has the smallest maximum loss. Bayes rule selects the action which has the smallest expected loss (risk); i.e., calculate the expected loss for each action a_j

$$\mathrm{E}[\mathcal{L}(\theta, a_j)] = \sum_{i=1}^{p} \Pr(\theta_i)\mathcal{L}(\theta_i, a_j)$$

A more thorough approach is to compare the sampling distributions of the losses under each action. For example, draw the histogram of losses under each action and compare not only the expected loss but look also at the variation in possible losses.

Example

For example, suppose the problem is to choose one of 2 management plans with respect to a single coho stock. The first plan is more conservative in terms of total harvest than the second plan. There are 3 possible states of nature, the initial survival rates, 1%, 3%, and 5% with probabilities 25%, 50%, and 25%, respectively. The following loss table is constructed with losses based on a combination of the squared difference between predicted escapement and desired escapement and some measure of economic loss (due to 'overescapement').

		Actions	
States	Probab.	Plan 1	Plan 2
Survival=1%	25%	5,000	10,000
Survival=3%	50%	3,000	4,000
Survival=5%	25%	7,000	3,000

The minimax choice would be Plan 1 (its maximum loss is 7,000).

For the Bayesian rule, first calculate the expected losses under both plans:

$$\begin{aligned}
\mathrm{E}[\mathcal{L}(\theta, \text{Plan 1})] &= (0.25 \times 5000) + (0.50 \times 3000) + (0.25 \times 7000) = 4500 \\
\mathrm{E}[\mathcal{L}(\theta, \text{Plan 2})] &= (0.25 \times 10,000) + (0.50 \times 4000) + (0.25 \times 3000) = 5250
\end{aligned}$$

The Bayes rule also picks Plan 1.

2 Three difficult issues

2.1 Choosing loss functions

Perhaps the greatest difficulty in carrying this approach out to evaluate pre-season management plans will be to define the loss function. This could be the most contentious issue at any rate. This is directly related to the variety of management objectives Jim Norris discussed at the March 16 meeting. Some examples:

1. Target escapement levels
2. Catch quotas per fishery
3. Catch allocation between 'user' groups (US:Canada, tribal:nontribal, commercial:recreational)

One simple approach is to define multiple loss functions for each of the above objectives and evaluate management plans with respect to each loss function. A more complicated, but tidier approach is to develop a loss function that is a weighted combination of the different losses. For example, suppose there are k stocks and m fisheries and the two objectives are achieving a particular escapement ($Esc.Targ$) and ensuring a particular total catch for each fishery ($Catch.Targ$). Let the individual loss components be defined as:

$$\begin{aligned}\mathcal{L}_1(\theta_i, a_j) &= \sum_{r=1}^{k} \frac{(Esc_r(\theta_i, a_j) - Esc.Targ)^2}{Esc.Targ \times k} \\ \mathcal{L}_2(\theta_i, a_j) &= \sum_{s=1}^{m} \frac{(Catch_s(\theta_i, a_j) - Catch.Targ)^2}{Catch.Targ \times m}\end{aligned}$$

The reason for the divisors in each loss function is to scale the measures somewhat equally given possibly different orders of magnitude. Then the combined loss function:

$$\mathcal{L}(\theta_i, a_j) = \lambda\mathcal{L}_1(\theta_i, a_j) + (1-\lambda)\mathcal{L}_2(\theta_i, a_j) \tag{1}$$

where the managers would have to choose λ.

2.2 High dimension to the uncertainty

A second difficulty is that the state of nature is very high dimensional, i.e., θ_i is really ($\theta_{i,1}$, $\theta_{i,2}$, ..., $\theta_{i,200}$), say. And each component, $\theta_{i,j}$, is often a continuous variable. For example, considering just a single coho stock and assuming the current hierarchical SSM is 'correct', at least the following uncertainties are present:

- Initial survival rate, $\gamma_{i,s}$
- Initial spatial distribution, $\gamma_{i,\alpha}$, $\gamma_{i,\beta}$
- Fishery catchability coefficients, $\gamma_{q,U.S.}$ and $\gamma_{q,Canada}$
- Migration parameters, $\gamma_{m,\alpha}$ and $\gamma_{m,\beta}$
- Release number
- Actual fishing effort (compared to predicted or planned)
- 'Natural' variation in the state and observation processes, the random noise w_t and v_t in the following:

$$\begin{aligned}n_t &= M_t S_t n_{t-1} + w_t \\ c_t &= H_t n_t + v_t\end{aligned}$$

I believe that this can be dealt with somewhat reasonably by simulation. Given a particular management plan, simulate the plan N times, calculate the loss function (equation (1), and draw a histogram of the losses.

2.3 Infinite number of actions

A third difficulty is that there are in fact an infinite number of management plans and the plans themselves can involve a complicated array of actions, including things like size limits, openings and closings, species-specific fisheries, catch-and-release of unmarked fish. Given the SSM uses effort as an input variable, I recommend (as Jim Norris did) that all management actions be translated into effort matrices. This will involve a pre-program of sorts– but will somewhat resemble the effort scaling factors currently used in various models.

Another problem though, also raised by Jim Norris, is that it is impossible to write down all possible effort matrices, and tedious to even write down a large set of possibilities. Instead, the manager would like an optimization (or near optimization) program to sift through the set of possible effort matrices[1]. A possible solution may be to use *simulated annealing.* Let a be a particular collection of effort matrices, i.e., the action or management plan, and define some initial plan a_0. The algorithm for step t is as follows:

Step 1 : Simulate the plan N times with a_t as the input and calculate the loss function (equation 1) for each simulation

Step 2 : Calculate the average (expected) loss and let this be the objective function to be minimized:

$$OF(a_t) \quad = \quad \frac{1}{N}\sum_{i=1}^{N}\sum_{i=1}^{N}\mathcal{L}(\theta *_i, a_t)$$

Step 3 : Randomly perturb a_t 'slightly' to get a_{t+1} (e.g., add Poisson random variables to each component of the effort matrix) and repeat Steps 1 and 2 using a_1 now.

Step 4 : Compare the two objective functions:

- If $OF(a_{t+1}) < OF(a_t)$, keep a_{t+1} and go back to Step 3 to get a_{t+2}
- Else keep a_{t+1} with probability

$$\exp\left[-\frac{OF(a_{t+1}) - OF(a_t)}{T}\right]$$

otherwise set $a_{t+1} = a_t$ (keep a_t)

Step 5 : if so many interations have been done, shrink T to kT.

Step 6 : Quit when 'close enough'.

[1]Note- given a particular effort matrix, there's a need for a post-program perhaps to convert the effort matrix into particular management actions?